

# Alternative models for QTL detection in livestock.

## I. General introduction

Jean-Michel Elsen<sup>a\*</sup>, Brigitte Mangin<sup>b</sup>, Bruno Goffinet<sup>b</sup>,  
Didier Boichard<sup>c</sup>, Pascale Le Roy<sup>c</sup>

<sup>a</sup> Station d'amélioration génétique des animaux, Institut national de la recherche agronomique, BP27, 31326 Auzeville, France

<sup>b</sup> Laboratoire de biométrie et d'intelligence artificielle, Institut national de la recherche agronomique, BP27, 31326 Auzeville, France

<sup>c</sup> Station de génétique quantitative et appliquée, Institut national de la recherche agronomique, 78352 Jouy-en-Josas, France

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**Abstract** – In a series of papers, alternative models for QTL detection in livestock are proposed and their properties evaluated using simulations. This first paper describes the basic model used, applied to independent half-sib families, with marker phenotypes measured for a two or three generation pedigree and quantitative trait phenotypes measured only for the last generation. Hypotheses are given and the formulae for calculating the likelihood are fully described. Different alternatives to this basic model were studied, including variation in the performance modelling and consideration of full-sib families. Their main features are discussed here and their influence on the result illustrated by means of a numerical example. © Inra/Elsevier, Paris

**QTL detection / maximum likelihood**

**Résumé** – Modèles alternatifs pour la détection de QTL dans les populations animales. **I. Introduction générale.** Dans une série d'articles scientifiques, des modèles alternatifs pour la détection de QTLs chez les animaux de ferme sont proposés et leurs propriétés sont évaluées par simulation. Ce premier article décrit le modèle de base utilisé, qui concerne des familles indépendantes de demi-germains de père, avec des phénotypes marqueurs mesurés sur deux ou trois générations et des phénotypes quantitatifs mesurés seulement sur la dernière génération. Les hypothèses sont données et l'expression de la vraisemblance décrite en détail. À partir de ce modèle de base, différentes alternatives ont été étudiées, incluant diverses modélisations des performances et la prise en compte de structures familiales avec de vrais ger-

\* Correspondence and reprints  
E-mail: elsen@toulouse.inra.fr

mains. Leurs principales caractéristiques sont décrites et une illustration est donnée.  
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**détection de QTL / maximum de vraisemblance**

## 1. INTRODUCTION

Over the last 15 years, tremendous progress has been achieved in genome analysis techniques leading to significant development of gene mapping in plant and animal species. These maps are powerful tools for QTL detection. The general principle for detecting QTL is that, within family (half-sibs, full-sibs or, when available, F2 or backcrosses from homozygous parental lines), due to genetic linkage, an association is expected between chromosomal segments received by progenies from a common parent and performance trait distribution, if a QTL influencing the trait is located within or close to the traced segment [24, 28]. Experiments were designed to identify QTL in major livestock species and the first QTLs have now been published for cattle [7] and pigs [1].

Following the early paper by Neimann-Sørensen and Robertson [22], the first statistical methods used to analyse these experiments considered only one marker at a time and were based on the analysis of variance of data including a fixed effect for the marker nested within sire (the two levels of this effect corresponding to the two alleles at a given locus which a given sire could transmit to its progeny). Efforts were made to better exploit available information in order to increase the power of detecting QTL and estimation behaviour.

- A better identification of grandparental chromosome segments transmitted by the parent was achieved using interval mapping [17] and further, for inbred and outbred populations, accounting for all marker information on the corresponding chromosome [10, 11, 13].

- Because the within-sire allele trait distribution is a mixture due to QTL segregation in the dam population, detection tests based on a comparison of likelihoods, were proposed to use data more thoroughly [14, 18, 27]. Intermediate approaches combining linear analysis of variance and exact maximum likelihood were also suggested to decrease the amount of computing required [15].

- While the first models considered families as independent sets of data, recent papers have shown how to include pedigree structure [9].

- The problem of testing for more than one QTL segregating on a chromosome has been dealt with by different authors in the simpler plant situation [12] but no final conclusions have yet been reached, in particular due to the lack of theory concerning the rejection threshold when testing in this multi-QTL context, as compared to the single QTL case [17, 23].

In developing software for analysing data from QTL detection designs in livestock, we started from a model similar to the one proposed by Knott et al. [15] and Elsen et al. [4] and compared alternative solutions for the estimation of phases in the sires, simplification of the likelihood, genetic hypotheses concerning the QTL and an extension of the methods to include the case of two QTLs and a mixture of full- and half-sib families. These comparisons and extensions will be published in related papers [8, 19, 20]. In this first

part, common hypotheses and notations are given, as well as the argument for the alternative studied. A numerical application illustrates how different conclusions may depend on the solution chosen.

## 2. BASIC MODEL

### 2.1. Hypotheses, notation

The population is considered as a set of independent sire families, all dams being themselves unrelated to each other and to the sires. Let  $i$  be the identification of a family. Thus, the global likelihood  $\Lambda$  is the product of within-sire likelihoods  $\Lambda_i$ .

Let  $ij$  be a mate ( $j = 1, \dots, n_i$ ) of sire  $i$  ( $i = 1, \dots, n$ ) and  $ijk$  ( $k = 1, \dots, n_{ij}$ ) the progeny of dam  $ij$ . Available information consists of individual phenotypes  $yp_{ijk}$  of progeny  $ijk$  for a quantitative trait and marker phenotypes of progeny, parents and grandparents for a set of codominant loci. Marker phenotypes will be denoted as follows:

sire $i$	$ms_i = \{ms_i^{l1}, ms_i^{l2}\}_{l=1, \dots, L}$
dam $ij$	$md_{ij} = \{md_{ij}^{l1}, md_{ij}^{l2}\}_{l=1, \dots, L}$
progeny $ijk$	$mp_{ijk} = \{mp_{ijk}^{l1}, mp_{ijk}^{l2}\}_{l=1, \dots, L}$
paternal grandsire $i$	$mss_i = \{mss_i^{l1}, mss_i^{l2}\}_{l=1, \dots, L}$
paternal granddam $i$	$mds_i = \{mds_i^{l1}, mds_i^{l2}\}_{l=1, \dots, L}$
maternal grandsire $ij$	$msd_{ij} = \{msd_{ij}^{l1}, msd_{ij}^{l2}\}_{l=1, \dots, L}$
maternal granddam $ij$	$mdd_{ij} = \{mdd_{ij}^{l1}, mdd_{ij}^{l2}\}_{l=1, \dots, L}$

Each pair (e.g.  $ms_i^{l1}, ms_i^{l2}$ ) corresponds to the two alleles observed at locus  $l$ .

When considering strictly half-sib families, only one progeny is measured per dam ( $n_{ij} = 1$ ), and the  $k$  index can be omitted.

Marker information concerning sire  $i$  family is pooled in vector  $M_i$  which includes at least the progeny phenotypes  $mp_{ijk}$ . Marker information concerning sire  $i$  progeny and sire  $i$  mates will be denoted  $mp_i$  and  $md_i$ , respectively. The vector of marker information concerning progeny of dam  $ij$  will be noted  $mp_{ij}$ . The vector of information concerning parents of sire  $i$  will be denoted  $mas_i = (mss_i, mds_i)$

$L$  marker loci belonging to a previously known linkage group are considered simultaneously. Recombination rates between marker loci are assumed to be known perfectly from previous independent analyses. A given marker locus within a linkage group is indexed as  $l$ .

In the multi-marker phenotypes  $ms_i$  and  $md_{ij}$ , the numbering of alleles  $\{1, 2\}$  for each locus is arbitrarily defined. These multi-marker phenotypes may have different corresponding genotypes  $hs_i$  and  $hd_{ij}$  with a given distribution of alleles on the two chromosomes.  $hs_i$  is an  $L \times 2$  matrix  $\{hs_i^1, hs_i^2\}$ , with the first column  $hs_i^1$  corresponding to the chromosome transmitted by the grandsire to the sire, and the second column  $hs_i^2$  corresponding to the chromosome transmitted by the granddam to the sire. Equivalently,  $hd_{ij} = \{hd_{ij}^1, hd_{ij}^2\}$ .

When available, the ancestry information concerning the markers ( $mss_i$  and  $mds_i$  for the sire  $i$ ) may help determine the phase, i.e. determining the grandparental origin of alleles  $ms_i^{l1}$  and  $ms_i^{l2}$ . Similarly,  $msd_{ij}$  and  $mdd_{ij}$  may provide information on the dam  $ij$  phase. This is not always possible, and ancestry information is not always available. Under these circumstances, the  $hs_i$  (and  $hd_{ij}$ ) genotypes are only given as a probability, using information from the progeny and, when collected, from the mates. The algebra for computing this probability is described in detail in the next section.

The position of locus  $l$  is given by  $x_l$ , its distance in cM from the extremity of its linkage group. At any position  $x$  within this group, the hypothesis is tested that sire  $i$  (in half-sib structure) or sire  $i$  and/or dam  $ij$  (in mixed half/full-sib structure) are heterozygous for a quantitative gene,  $QTL_x$ , influencing the mean of the trait distribution. In the case of half-sib families, this mean is  $\mu_i^{x1}$  or  $\mu_i^{x2}$ , depending on the grandparental segment 1 or 2 received from the sire at location  $x$ . In the case of full-sib families, this mean is  $\mu_{ij}^{x11}$ ,  $\mu_{ij}^{x12}$ ,  $\mu_{ij}^{x21}$  or  $\mu_{ij}^{x22}$ , depending on the grandparental segments 1 or 2 received from the sire and dam.

Given the sire allele received at location  $x$  ( $d_{ijk}^x = 1$  or  $2$ ), or in full-sib families, given the sire and dam alleles received ( $d_{ijk}^x = (1, 1), (1, 2), (2, 1)$  or  $(2, 2)$ ), the quantitative trait for progeny  $ijk$  is normally distributed with a mean  $\mu_i^{x d_{ijk}^x} + X_{ijk}\beta$  and a variance  $\sigma_e^2$ ,  $\beta$  being a vector of fixed effects and  $X_{ijk}$  the corresponding incidence vector.

In the following, the description is restricted to the half-sib family structure and the  $\beta$  vector is omitted. An extension to include a mixed structure with full- and half-sib families is described in Le Roy et al. [19].

## 2.2. Likelihood

With the hypotheses described above, and omitting the  $k$  indices, the likelihood is

$$\Lambda^x = \prod_i \Lambda_i^x = \prod_i \sum_{hs_i} p(hs_i/M_i) \prod_{j=1}^{n_i} \sum_{q=1}^2 p(d_{ij}^x = q/hs_i, M_i) f(yp_{ij}/d_{ij}^x = q)$$

This likelihood depends on the following three terms.

1) The penetrance function  $f(yp_{ij}/d_{ij}^x = q)$  which is conditional on the  $q$  chromosome segment transmitted by the sire. This penetrance will be assumed to be normal. Let  $\phi(y; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{-\frac{1}{2} \left(\frac{y - \mu}{\sigma}\right)^2\right\}$ . This gives  $f(yp_{ij}/d_{ij}^x = q) = \phi(yp_{ij}; \mu_i^{xq}, \sigma_e^2)$ .

When necessary, the following alternative parameterization will be used for the mean:  $\mu_i^{x1} = \mu_i + \alpha_i^x/2$  and  $\mu_i^{x2} = \mu_i - \alpha_i^x/2$ ,  $\alpha_i^x$  being the within half-sib average effect of the QTL substitution, denoted as the QTL substitution effect below. In the particular situation where the QTL has two isofrequent alleles with an additive effect, the expected effect  $\alpha_i^x$  at the exact location of the QTL is equal to half the genetic difference between homozygous carriers [6].

It must be emphasized that the half-sib correlation is accounted for by estimating within-sire means  $\mu_i^{xq}$  or  $\mu_i$ . In Knott et al. [15], the deviation from the family mean  $yp_{ij} - \sum_j yp_{ij}/n_i$  was considered rather than  $yp_{ij}$  directly,

with some approximation to simplify the likelihood computation [4]. All these approaches assumed no relationship between parents in the pedigree.

2) The transmission probability  $p(d_{ij}^x = q/h_{si}, M_i)$ , i.e. probability for progeny  $ij$  that it received from its sire the  $q$ th chromosome segment including location  $x$  ( $q = 1$  from the grandsire,  $q = 2$  from the granddam).

Let  $\gamma_{ij}^l(h_{si})$  be a variable indicating the grandparental origin of marker  $l$  for progeny  $ij$  (0 unknown, 1 grand sire, 2 grand dam). Let A, B and C be three possible marker alleles for locus  $l$ .  $\gamma_{ij}^l(h_{si})$  is computed as follows

		if		then
$hs_i^1$	$hs_i^2$	$md_{ij}$	$mp_{ij}$	$\gamma_{ij}^l(h_{si})$
A	A	$\forall$	AC ( $\forall$ C)	0
A	B	$\forall$	AC ( $\forall$ C $\neq$ B)	1
A	B	$\forall$	BC ( $\forall$ C $\neq$ A)	2
A	B	BC ( $\forall$ C $\neq$ A)	AB	1
A	B	AC ( $\forall$ C $\neq$ B)	AB	2
A	B	AB	AB	0
A	B	unknown	AB	0

For progeny  $ij$ , let  $l_l$  and  $l_r$  be the closest flanking informative marker loci to  $x \in [l, l+1]$  (with  $\gamma_{ij}^{l_l}(h_{si}) \neq 0$  and  $\gamma_{ij}^{l_r}(h_{si}) \neq 0$ ):  $l_l \leq l \leq x \leq l+1 \leq l_r$ . The recombination rate  $r(l_a, l_b)$  between marker loci  $l_a$  and  $l_b$  may be computed using a map function. Absence of interference was hypothesized, allowing use of the Haldane function. In this case  $r(l_a, l_b) = \frac{1}{2}(1 - \exp\{-2|x_{l_b} - x_{l_a}|\})$ .

$p(d_{ij}^x = q/h_{si}, M_i)$  is then computed as follows

case	$p(d_{ij}^x = q/h_{si}, M_i)$
$q = \gamma_{ij}^{l_l}(h_{si})$ and $\gamma_{ij}^{l_l}(h_{si}) = \gamma_{ij}^{l_r}(h_{si})$	$\frac{(1 - r(l_l, x))(1 - r(l_r, x))}{1 - r(l_l, l_r)}$
$q \neq \gamma_{ij}^{l_l}(h_{si})$ and $\gamma_{ij}^{l_l}(h_{si}) = \gamma_{ij}^{l_r}(h_{si})$	$\frac{r(l_l, x)r(l_r, x)}{1 - r(l_l, l_r)}$
$q = \gamma_{ij}^{l_l}(h_{si})$ and $\gamma_{ij}^{l_l}(h_{si}) \neq \gamma_{ij}^{l_r}(h_{si})$	$\frac{(1 - r(l_l, x))r(l_r, x)}{r(l_l, l_r)}$
$q = \gamma_{ij}^{l_r}(h_{si})$ and $\gamma_{ij}^{l_l}(h_{si}) \neq \gamma_{ij}^{l_r}(h_{si})$	$\frac{r(l_l, x)(1 - r(l_r, x))}{r(l_l, l_r)}$

The first case corresponds to the absence of recombination between flanking markers, the second and third to one recombination (on the left or on the right of the QTL) and the forth to a double recombination situation, on the left and on the right of the QTL.

3) The genotype probability conditional on the marker information  $p(hs_i/M_i)$ . In the case of half-sib families, marker information  $M_i$  is  $M_i = (mas_i, ms_i, md_i, mp_i)$ . Genotype probabilities were computed from the relation:

$$p(hs_i/M_i) = \frac{p(mp_i/hs_i, md_i).p(hs_i/mas_i, ms_i)}{\sum_{hs_i} p(mp_i/hs_i, md_i).p(hs_i/mas_i, ms_i)}$$

–  $p(hs_i/mas_i, ms_i)$  was computed considering successively each marker locus. For a single locus  $l$ , with alleles A, B, C and D (or O when not measured), possible values for the sire genotype were deduced from phenotypic information as described in *table I*.

**Table I.** Possible sire genotype depending on the phenotypes of the sire itself and of its parents.

Case	$ms_i$	$mss_i$	$mds_i$	$hs_i$
1	AA	∅	∅	A/A
2	AB	AC(∅C ≠ B)	∅	A/B
3	AB	BC(∅C ≠ A)	∅	B/A
4	AB	∅	AC(∅C ≠ B)	B/A
5	AB	∅	BC(∅C ≠ A)	A/B
6	AB	AB, O	AB, O	A/B, B/A
7	O	AA	BB(∅B)	A/B
8	O	AB	CC(∅C)	A/C, B/C
9	O	AB	CD	A/C, A/D, B/C, B/D
10	O	O	AA	–/A
11	O	O	AB	–/A, –/B
12	O	AA	O	A/–
13	O	AB	O	A/–, B/–
14	O	O	O	–/–

– any existing allele.

$p(hs_i/mas_i, ms_i) = 0$  or  $1$  in the first five cases and case 7,  $0$  or  $1/2$  for cases 6 and 8,  $0$  or  $1/4$  for case 9. In the other situations, (10–13), this probability depends on the allele frequencies of marker  $l$  (in some instances, the genotype of a sire without individual measurement at locus  $l$  may be partially rebuilt from the progeny information: a sire with at least one progeny AA or one progeny AC from a dam CD is known to carry the A allele; a sire known to carry both A and B alleles will have, with a probability of  $1/2$ , either A/B or B/A genotypes at this locus).

In practice, the exploration of possible genotypes was restricted first by assuming linkage equilibrium between marker loci, second by not using marker information in cases where the probability depends on allele frequencies.

In the particular case where no information is available on the ancestry,  $p(hs_i/ms_i) = (\frac{1}{2})^{L_i}, \forall hs_i$  consistent with  $ms_i$ ,  $L_i$  being the number of heterozygous marker loci for sire  $i$  and

$$p(hs_i/M_i) = \frac{p(mp_i/hs_i, md_i)}{\sum_{hs_i} p(mp_i/hs_i, md_i)}$$

considering, in the summation, only those  $hs_i$  which are consistent with  $ms_i$ .

–  $p(mp_i/hs_i, md_i)$ . Within sire genotype and dam marker phenotype, progeny marker phenotypes are independent, giving

$$p(mp_i/hs_i, md_i) = \prod_j p(mp_{ij}/hs_i, md_{ij})$$

The probability for progeny  $ij$  may be computed using the  $t_{ij}$  vectors of possible transmission from its sire  $i$ :  $t_{ij} = (t_{ij}^1, t_{ij}^2, \dots, t_{ij}^{L_i})$ , which depends on the  $\gamma_{ij}^l(hs_i)$  indicators ( $t_{ij}^l = 1$  if  $\gamma_{ij}^l(hs_i) = 1$ ,  $t_{ij}^l = 2$  if  $\gamma_{ij}^l(hs_i) = 2$ ,  $t_{ij}^l = 1$  or 2 if  $\gamma_{ij}^l(hs_i) = 0$ ).

The following recurrence was used to obtain  $p(mp_{ij}/hs_i, md_{ij})$ :

$$p(mp_{ij}/hs_i, md_{ij}) = \sum_{t_{ij}^{L_i}} \psi(t_{ij}^{L_i})$$

with

$$\psi(t_{ij}^l) = p(mp_{ij}^l/t_{ij}^l, hs_i, md_{ij}) \sum_{t_{ij}^{l-1}} p(t_{ij}^l/t_{ij}^{l-1}) \psi(t_{ij}^{l-1})$$

and

$$\psi(t_{ij}^1) = p(mp_{ij}^1/t_{ij}^1, hs_i, md_{ij})$$

Elements of this recurrence are  $p(t_{ij}^l/t_{ij}^{l-1})$  which is simply  $1 - r(l-1, l)$  if  $t_{ij}^l = t_{ij}^{l-1}$  and  $r(l-1, l)$  if  $t_{ij}^l \neq t_{ij}^{l-1}$ , and  $p(mp_{ij}^l/t_{ij}^l, hs_i, md_{ij})$  which is 0, 1/2 or 1 when  $md_{ij}$  was measured, the frequency, in the dam population, of the allele which was not given by the sire in  $mp_{ij}^l$  when  $md_{ij}^l$  was not measured.

To avoid inaccurate estimation of these frequencies, we only considered, for each sire family, marker loci for which the paternal transmission was certain ( $\gamma_{ij}^l \neq 0$ ). With this restriction, only one  $t_{ij}^l$  is possible for each locus, and  $p(mp_{ij}/hs_i, md_{ij}) = \prod_l p(mp_{ij}^l/t_{ij}^l, hs_i, md_{ij}) p(t_{ij}^l/t_{ij}^{l-1})$ . It follows that the dam allele frequencies disappear when computing the ratio

$$\frac{p(mp_i/hs_i, md_i) \cdot p(hs_i/ms_i, ms_i)}{\sum_{hs_i} p(mp_i/hs_i, md_i) \cdot p(hs_i/ms_i, ms_i)}$$

### 3. ALTERNATIVE MODELS

The preceding model was close to the models proposed by Georges et al. [7], Knott et al. [15] and Elsen et al. [4] when searching for QTL in similar

populations. In related papers [8, 19, 21] alternatives to this model will be explored, dealing with the computation of genotype probabilities, the choice of the genetic model and the study of mixed half- and full-sib families. After a brief description of these extensions, a numerical application will illustrate their properties.

### 3.1. Rationale for the alternatives studied

#### 3.1.1. Sire genotype estimates

In the full model described above, all possible genotypes for the sire  $i$  were successively considered, the likelihood  $\Lambda_i$  being a weighted sum of likelihoods conditional on these genotypes  $hs_i$ . This may be very time consuming for large linkage groups, since a maximum of  $2^L$  sire genotypes is possible. Another option could be to limit the explored sire genotypes to the most probable one a priori, comparing the  $p(hs_i/M_i)$ . In Knott et al. [15], only the most probable sire genotype was considered, its probability being estimated in a simplified way. Alternatively, the sire genotype could itself be considered as a variable to be optimized as are the means and variances. In our application, the genotype  $\widetilde{hs_i}$  should be attributed to sire  $i$  if

$$\widetilde{hs_i} = \text{Argmax}_{hs_i} \left( \prod_{j=1}^{n_i} \sum_{q=1}^2 p(d_{ij}^x = q/hs_i, M_i) f(y p_{ij}/d_{ij}^x) \right)$$

This is the way mixtures are considered in the classification likelihood approach [20]: no credit is given to prior information on sire genotypes (a position which could be justified by a lack of credibility of needed hypotheses concerning for instance linkage equilibrium between marker loci).

Not to be so extreme, we suggest considering only the most a priori probable genotype  $hs_i$  in this optimization of  $\Lambda_i$  in  $hs_i$  (practically, to restrict the domain of  $hs_i$  to genotypes with  $\text{prob}(hs_i/M_i)$  higher than a minimum value).

Finally, an intermediate solution could be the maximization of the joint likelihood of sire genotype  $hs_i$  and observations  $yp_{ij}$ :

$$\prod_i p(hs_i/M_i) \prod_{j=1}^{n_i} \sum_{q=1}^2 p(d_{ij}^x = q/hs_i, M_i) f(y p_{ij}/d_{ij}^x)$$

These options were compared by Mangin et al. [21].

#### 3.1.2. Linear approximation of the likelihood

Within sire genotype, the offspring trait distribution was described as a mixture of normal distributions, weighted by the transmission probabilities  $p(d_{ij}^x = q/hs_i, M_i)$ . For relatively small QTL effect, differences in the means of these normal distributions are not expected to be high, and linearization has

been suggested. It is supposed that  $\sum_{q=1}^2 p(d_{ij}^x = q/hs_i, M_i) \phi(y p_{ij}; \mu_i^{xq}, \sigma_e^2)$  is



close to  $\phi(y_{pij}; \sum_{q=1}^2 p(d_{ij}^x = q/h s_i, M_i) \mu_i^{xq}, \sigma_e^2)$ . The efficiency of this linearization has been studied by Mangin et al. [21].

### 3.1.3. Modelling QTL allele distribution

In the basic model, all sires are assumed to be heterozygous for a QTL at location  $x$ , with trait means in the daughter  $\mu_i^{x d_{ij}^x}$  depending on the  $d_{ij}^x$  allele received from her sire. Twice the number of sire means thus have to be estimated. Two different genetic hypotheses were studied by Goffinet et al. [8] in which the QTL effect was considered to be random.

The first modelling assumed that the QTL effect is normally distributed  $\mathcal{N}(0, \sigma_a^2)$ , with only one parameter ( $\sigma_a^2$ ) being estimated, potentially increasing the QTL detection power. This global approach to the sire population is probably more justified when the number of sire families is high, since the sample of sire families is representative of the whole population of sires.

The second modelling assumed that two alleles only are segregating at the QTL. This situation is often hypothesized when testing for the existence of a major gene (e.g. [5]). The most important feature of this modelling is the across-family estimation of the QTL allele effects, which makes maximization of the likelihood more complex ( $\Lambda_i$  are no longer independent) but increases the power of the test in some cases.

### 3.1.4. Heteroskedasticity

There are different arguments in favour of non-equality of within QTL variance ( $\sigma_e^2$ ) between families. The most important is probably the non-identity of allele distributions at other QTLs than the tested one, in particular if some of them have major effects on the trait. To increase the robustness of the method, a heteroskedastic model was studied by Goffinet et al. [8], considering within-sire family variances  $\sigma_{ei}^2$ .

### 3.1.5. Full-sib families

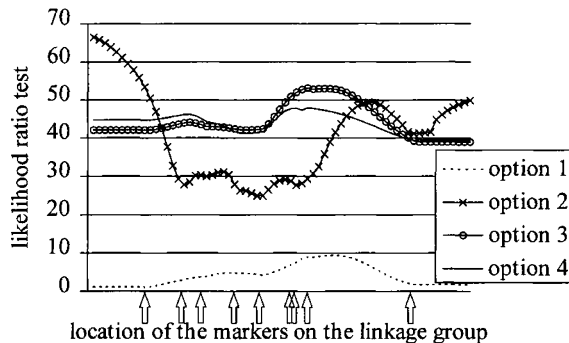
As already mentioned, the generalization of our approach to populations mixing half- and full-sib families was proposed by Le Roy et al. [19]. In their modelling, the global population is a set of independent sire families, each sire being mated to independent dams having more than one progeny. This is a simple representation of populations used for QTL detection in pigs [1].

## 3.2. Example

QTLMAP, a program written in FORTRAN, considers all the previous alternatives. It is available on request. Inputs for this program include pedigree information, marker and quantitative genotypes of studied half- or full-sib families of the population, and the marker map assumed to be perfectly known from previous analyses. Outputs are basic statistics on the case studied, profile likelihoods along the explored linkage groups for different options concerning the hypotheses, as described above.

As an illustration, here are the results of a study organized within the framework of a European network (CT940508) and discussed during two international seminars on QTL detection methods (workshops held in Liege and Nouzilly in 1996 and the 1996 ISAG meeting, respectively). A summary of the last meeting was published by Bovenhuis et al. [2].

The granddaughter design for QTL detection in dairy cattle consisted of 20 sire families. The linkage group comprised nine marker loci from the bovine chromosome 6, located at positions 0, 13, 20, 31, 41, 52, 54, 58 and 95 cM. Ten sets of quantitative phenotypes were given, five being simulated, five corresponding to real data collected in the granddaughter design. A detailed description of the data is given by Spelman et al. [25]. An example of analysis is shown in *figure 1* for trait 4, using different options of our software. In all these options the only sire genotype considered is the most probable a priori, from  $p(hs_i/M_i)$ . Option 1 is based on a prior normal distribution of the QTL effect while in other options within-sire QTL effects ( $\alpha_i^x$ ) are estimated without prior information on their distribution. Option 2 is based on the full within  $hs_i$  likelihood but other options considered the linear approximation. The within QTL variance is unique in options 2 and 4, and depends on the sire in options 1 and 3. The low values of the option 1 likelihood ratio test are linked to the limited number of QTL effects (1:  $\sigma_a^x$  versus 20:  $\alpha_i^x$ ) estimated in this case. The likelihood profiles suggest a QTL between markers 6 and 9 in the linear versions, with flat, non-informative, tails. The non-linear version behaves quite differently with a shift of the maximum towards the right side (between markers 8 and 9) of the linkage group and bumps at the extremities.



**Figure 1.** Profile likelihoods for four of the methods studied. 1: Linear, heteroskedastic, normal prior for the QTL effects; 2: non-linear, homoskedastic, no prior; 3: linear, heteroskedastic, no prior; 4: linear, homoskedastic, no prior.

#### 4. CONCLUSION

The main features of the models and test statistics we compared have been discussed in this introduction. The companion papers [8, 19, 21] relate to these comparisons in detail. Our approach, and its corresponding software, have limitations which should be overcome in the future.

Parents (sires, dams and possibly grandparents) were assumed to be unrelated. This may not be essential since the QTL detection is mostly based on within-family analyses. However, this could cause some loss of power for the test and of precision for parameter estimation, in particular when considering a distribution for the QTL effects. Grignola et al. [9] for instance accounted for such genetic relationships between parents in their modelling. A numerical comparison of these descriptions is necessary.

Our model is parametric, assuming normality of the penetrance function. This is probably general, but could be invalid when a major QTL is segregating independently of the studied linkage group or when the trait is clearly non-normal (discrete or all-or-none trait). A non-parametric approach was proposed by Kruglyak and Lander [16] and was generalized by Coppieters et al. [3] with an application to the set of data we used here. It might be helpful to merge its characteristics with the genetic part of our model.

We have not used all the information on marker allele transmission: only unambiguous segregation information was included in our likelihood. A more exhaustive use of this information has been proposed, using Monte Carlo Markov chain [26]. Such an approach is very computationally demanding and, again, a numerical comparison of test power and estimation precision should be made to assess the respective usefulness of both approaches.

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